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On the Consistency of the Regularization of Gauge Theories by High Covariant Derivatives

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Abstract

We show that regularization of gauge theories by higher covariant derivatives and gauge invariant Pauli-Villars regulators is a consistent method if the Pauli-Villars vector fields are considered in a covariant α–gauge with $\alpha \neq 0$ and a given auxiliary preregularization is introduced in order to uniquely define the regularization. The limit $\alpha \to 0$ in the regulating Pauli-Villars fields is pathological and the original Slavnov proposal in covariant Landau gauge is not correct because of the appearance of massless modes in the regulators which do not decouple when the ultraviolet regulator is removed. In such a case the method does not correspond to the regularization of a pure gauge theory but that of a gauge theory in interaction with massless ghost fields. However, a minor modification of Slavnov method provides a consistent regularization even for such a case. The regularization that we introduce also solves the problem of overlapping divergences in a way similar to geometric regularization and yields the standard values of the β and γ functions of the renormalization group equations.

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1 Introduction

In spite of the success of dimensional regularization in perturbation theory, the existence of interesting non-perturbative phenomena in gauge theories requires the introduction of a non-perturbative regularization. Discretization of space-time leads in a natural way to lattice regularizations which preserve gauge invariance and have a non-perturbative meaning. Unfortunately, the method does not seem to be appropriate for the regularization of chiral, supersymmetric or topological theories. The construction of a non-perturbative gauge invariant regularization of gauge theories in a continuum space-time has been a challenging problem in gauge theories. A natural candidate has always been a gauge invariant generalization of Pauli-Villars methods involving high derivatives in the action.

However, the regularization of gauge theories by high covariant derivatives is plagued of difficulties. In scalar field theories the addition of a number of derivatives to the kinetic term of the action is enough to make the theory ultraviolet finite. In gauge theories it is well known that the generalization of such a method requires the introduction of covariant derivatives instead of ordinary derivatives to preserve gauge invariance [1]. Although higher loop diagrams acquire by power counting a negative superficial divergent dimension, the divergences of one loop diagrams are not smoothed by higher covariant derivatives [1] [2]. One way of getting rid of the remaining one loop divergences could be the introduction of an additional gauge invariant Pauli-Villars regularization [3] (see also [2]). The concrete implementation of such regularization introduced by Slavnov in Ref. [3] (see also Ref. [4] for a review) encounters, however, two problems. First, as it is well known the regularization does not remove all the ultraviolet divergences. In fact, diagrams with external Pauli-Villars lines are not regularized and their contributions to subdiagrams of higher loop diagrams with external gluon lines is divergent [5] [6]. The second problem has been pointed out by a calculation of the β and γ-functions of pure gauge theory using the Slavnov regularization method [7]. The result differs from the standard values [8] which are known to be universal [9] [1].

Any regularization method has to satisfy two main requirements. First, it has to make all Green functions of the theory finite. But this is not enough, it has to satisfy one further condition. The terms of the effective action that at a certain order in perturbation theory are finite in the original theory, should recover their exact finite values after the removal of regulating mass parameters. In particular, at a formal level, the regularized partition

[1] There is a third problem which is associated to the ambiguity introduced by the cancelation of one loop divergences into the definition of the regularization. Such an ambiguity, which is a general feature of any Pauli-Villars regularization, can be eliminated by a suitable choice of an auxiliary pre-regularization prescription or by a common notation prescription for all one-loop diagrams involving Pauli-Villars field propagators [10]

function must converge to the original one.

In this paper, we show that the method of regularization proposed in Ref. [3] is not a regularization of a pure Yang-Mills theory in this sense but rather that of Yang-Mills in interaction with a massless grassmannian ghost ξ and a real commuting one ϕ in the adjoint representation,

$$S(A) = \frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{\mu\nu}_a + \int d^4x\,(D^\mu\bar{\xi})_a(D_\mu\xi)^a + \frac{1}{2}\int d^4x\,(D^\mu\phi)_a(D_\mu\phi)^a$$

The existence of such a phenomenon is due to the fact that the Pauli-Villars counterterms introduced in Ref. [4] do not disappear when the ultraviolet cutoff is removed. This fact does not affect the one loop calculations of the renormalization of the Chern-Simons coupling constant using the Slavnov method, because the extra ghost interactions do not generate unphysical pseudoscalar radiative corrections.

It is fairly easy, however, to correct the form of the Pauli-Villars regulators to get a consistent regularization. The problem of overlapping divergences requires a more drastic modification of the standard prescription.

A solution of all these problems was introduced in Refs. [5][11] in terms of a geometric interpretation of the regularization method. Other consistent regularization methods with higher covariant derivatives which are not affected by those problems were introduced in Refs. [12][13].

In this paper, we discuss a simpler higher covariant derivative regularization method, closer to the original Slavnov proposal and free of any of the difficulties mentioned above. In section 2 we analyse the origin of the unphysical corrections in the standard higher covariant derivative regularization and we propose a method to overcome the problem. The advantages of the regularization by Pauli-Villars field in α-gauges are investigated in section 3. In section 4 we analyse the problem of overlapping divergences and show how it can be solved. In sections 5 and 6 we calculate the β and γ-functions of Yang-Mills theory at one loop in the new regularization scheme and compare the results with those obtained by geometric regularization. Finally, in section 7 we present the conclusions of our work.

2 High Covariant Derivatives Regularization

For simplicity we shall consider $SU(N)$ gauge theories, although the results can be straightforwardly extended to arbitrary gauge groups. The euclidean action of Yang-Mills theory is given by

$$S(A) = \frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{\mu\nu}_a$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$ is the field strength of the gauge field A^a_μ.

The high covariant derivatives method proposed by Slavnov [3] (see [4] for a review) proceeds by two steps. The Yang-Mills action is replaced by its regularized version

$$S_\Lambda(A) = \frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}[(I+\Delta_\lambda/\Lambda^2)^n]^{a'\mu}_{a\,\mu'}F^{\mu'\nu}_{a'}, \tag{2.1}$$

where Δ_λ is the covariant differential operator given by

$$(\Delta_\lambda)^{a\,\mu}_{a'\mu'} = -D^{2a}_{\ a'}\delta^\mu_{\mu'} + 2\lambda f^a_{a'c}F^{c\mu}_{\mu'}$$

in terms of the covariant derivative $D^a_{\mu b} = \partial_\mu\delta^a_b + f^a_{bc}A^c_\mu$ and λ is an arbitrary real constant.

The partition function for the regularized action in α_0–gauge is

$$\mathcal{Z} = \int\prod_x dA(x)\det\partial^\mu D_\mu\exp\left\{-S_\Lambda(A) - \frac{1}{2\alpha_0}\int d^4x\,\partial^\mu A^a_\mu(I-\partial^2/\Lambda^2)^n\partial^\nu A^a_\nu\right\},$$

where we have introduced the operator $(1-\partial^2/\Lambda^2)^n$ in the gauge fixing term in order to have an ultraviolet asymptotic behaviour for the longitudinal modes of the propagator similar to that of the transverse ones. In this way, provided $n\geq 2$, all 1PI diagrams with more than one loop acquire a negative degree of divergence by power counting [1]. However, the degree of divergence of one-loop 1PI diagrams is unchanged by the addition of covariant derivatives. In other words, the theory is not completely regularized by the simple fact of adding higher covariant derivatives to the action as for the case of scalar field theories [1][2].

Notice, however, that due to the regular behaviour of the gluonic propagator the contributions in the effective action to the ghost two point function and gluon-ghost vertex are finite at one loop. This implies that one loop divergences exclusively arise in diagrams with only external gluon lines, and are given by the following product of determinants

$$\mathcal{Z}_{\rm div} = \det(-\partial^\mu D_\mu)\det{}^{-1/2}\mathcal{Q} \tag{2.2}$$

with

$$\det{}^{-1/2}\mathcal{Q} = \int\prod_x dq(x)\,\exp\,\Big\{\,-\frac{1}{2}\int d^4x\,d^4y\,q^a_\mu(x)\frac{\delta^2 S_\Lambda}{\delta A^a_\mu(x)\delta A^b_\nu(y)}q^b_\nu(y) \\ -\frac{1}{2\alpha_0}\int dx\,\partial^\mu q^a_\mu(I-\partial^2/\Lambda^2)^n\partial^\nu q^a_\nu\Big\}.$$

On the other hand, since Faddeev-Popov ghost fields only get finite renormalizations at one loop, the divergences of $\mathcal{Z}$ can be written in a gauge invariant way. This was the main observation made by Slavnov [3] who proved that one loop divergences of Yang-Mills theory $\mathcal{Z}_{\rm div}$ are formally equal to those of

$$\mathcal{Z}_{\rm div} = \det(-D^2)\det{}^{-1/2}\mathcal{Q}^L_0 \tag{2.3}$$

where

$$\det{}^{-1/2}\mathcal{Q}_0^L = \int\prod_x dq(x)\,\delta(D^\mu q_\mu(x))\,\exp\left\{-\frac{1}{2}\int d^4x d^4y\, q_\mu^a(x)\frac{\delta^2 S_\Lambda}{\delta A_\mu^a(x)\delta A_\nu^b(y)}q_\nu^b(y)\right\}. \quad (2.4)$$

We remark that all the determinants in (2.3) are explicitly gauge invariant. This fact can be understood as a consequence of the absence of divergent radiative corrections to the interaction of Faddeev-Popov ghost fields, which also implies that the BRST symmetry is only renormalized by finite radiative corrections. Moreover, gauge invariance is not lost when we add mass terms in (2.3). Then, it seems natural to use these determinants as the Pauli-Villars counterterms that subtract divergences at one loop in a gauge invariant way. This is the Slavnov approach introduced in Ref. [3] where the author considers the Pauli-Villars regulator

$$\det{}^{-1/2}\mathcal{Q}_m = \det(\Lambda^2 m^2 - D^2)\det{}^{-1/2}\mathcal{Q}_m^L$$

with

$$\begin{aligned}\det{}^{-1/2}\mathcal{Q}_m^L = \int\prod_x dq(x)\;\delta(D^\mu q_\mu(x))\,\exp\Big\{&-\frac{1}{2}\int d^4x d^4y\, q_\mu^a(x)\frac{\delta^2 S_\Lambda}{\delta A_\mu^a(x)\delta A_\nu^b(y)}q_\nu^b(y)\\ &-\frac{1}{2}m^2\Lambda^2\int d^4x\, q^2\Big\}. \quad (2.5)\end{aligned}$$

The regularized partition function

$$\begin{aligned}\mathcal{Z}_\Lambda &= \int\prod_x dA(x)\exp\left\{-S_\Lambda(A) - \frac{1}{2\alpha_0}\int d^4x\,\partial^\mu A_\mu^a(I-\partial^2/\Lambda^2)^n\partial^\nu A_\nu^a\right\}\\ &\cdot\det\,(-\partial^\mu D_\mu)\prod_j \det{}^{-s_j/2}\mathcal{Q}_{m_j}, \quad (2.6)\end{aligned}$$

is, then, free of divergences at one loop provided the s_j parameters are chosen so that

$$\sum_j s_j + 1 = 0. \quad (2.7)$$

Note that Pauli-Villars conditions do not involve the masses as it is usually the case. This is due to gauge invariance and the high derivative terms in the action that make finite the terms depending on m.

Strictly speaking, in order to properly analyse the mechanism of cancellation of one-loop divergences it becomes necessary to introduce an auxiliary regulator to handle the different divergences which appear by power counting [13][5] [7]. There are several choices. Dimensional regularization has the advantage of preserving gauge invariance although it does not have a non-perturbative interpretation. If we consider such an auxiliary regulator the condition (2.7) is enough to make finite any one-loop n-point function with external

gluon lines (see appendix A for an explicit computation). In section 4 we shall discuss other possible choices of auxiliary regulator and the new conditions we must impose in order to have a finite theory.

We have analyzed so far the main issues of the high covariant derivatives regularization method proposed by Slavnov [3]. We shall now discuss the problems raised by this method. In particular, we shall see that it is not a suitable regularization of the theory.

Finiteness of Green functions is, of course, something that any regularization must satisfy but this is not enough. One further requirement is that the terms of the effective action that at a certain order in perturbation theory are finite in the original theory, should converge to the same value in the regularized one when the regulator Λ is removed. In particular at a formal level, the regularized partition function $\mathcal{Z}_{\rm reg}$ must converge to the original one $\mathcal{Z}$ as the cut-off parameter Λ goes to ∞. This requirement is not satisfied by the regularization presented in [3].

The problem is that Pauli-Villars determinants $\det\mathcal{Q}_m$ do not converge formally to a constant, as they should, when the cutoff is removed. In fact, we have that

$$\lim_{\Lambda\to\infty}\det{}^{-1/2}\mathcal{Q}_m = \int\prod_x dq(x)\,\delta(D^\mu q_\mu(x))\exp\left\{-\frac{1}{2}\int d^4x\,q^2(x)\right\} \tag{2.8}$$

that depends on A through the delta functional $\delta(D^\mu q_\mu)$. This can be seen more precisely if we perform in (2.8) the change of variables $q\to(q^\perp,\phi)$, where $q_\mu = q_\mu^\perp + D_\mu\phi$,

$$q_\mu^\perp = q_\mu - D_\mu D^{-2}D^\nu q_\nu \qquad \text{and} \qquad \phi = D^{-2}D^\nu q_\nu. \tag{2.9}$$

The Jacobian of the transformation is $\det^{1/2}(-D^2)$. Therefore

$$\lim_{\Lambda\to\infty}\det{}^{-1/2}\mathcal{Q}_m = \int\prod_x d\phi(x)\;dq^\perp(x)\;\delta(D^2\phi(x))\,\exp\left\{-\frac{1}{2}\int d^4x(q^\perp - D\phi)^2(x)\right\}$$
$$\cdot\det{}^{1/2}(-D^2),$$

and using the identity $\delta(D^2\phi)=\det^{-1}(-D^2)\,\delta(\phi)$, it reduces to

$$\lim_{\Lambda\to\infty}\det{}^{-1/2}\mathcal{Q}_m = \det{}^{-1/2}(-D^2),$$

i.e. there is a net contribution of the Pauli-Villars regulators to the effective action. This fact explains why computations of the β-function using this regularization [7] do not agree with the standard value obtained by well established regularization methods [8]. Using the Pauli-Villars condition (2.7) one obtains that the total anomalous contribution to the effective action at one loop is given by

$$\det{}^{1/2}(-D^2),$$

and it is easy to compute its contribution to the β-function

$$\Delta\beta(g) = -\frac{1}{6}\frac{g^3 N}{16\pi^2}.$$

If we add this contribution to the standard one

$$\beta(g) = -\frac{11}{3}\frac{g^3 N}{16\pi^2},$$

we get the anomalous result

$$\beta^{\mathrm{S}}(g) = -\frac{23}{6}\frac{g^3 N}{16\pi^2}$$

found in Ref. [7]. The same occurs for the γ_A coefficient of anomalous dimensions of the gauge fields. If we add the extra contribution coming from the determinant,

$$\Delta\gamma_A(g) = \frac{1}{6}\frac{g^2 N}{16\pi^2},$$

to the standard value in α_0 gauge

$$\gamma_A(g) = \left(\frac{13}{6} - \frac{\alpha_0}{2}\right)\frac{g^2 N}{16\pi^2},$$

we get the value

$$\gamma_A^{\mathrm{S}}(g) = \left(\frac{14}{6} - \frac{\alpha_0}{2}\right)\frac{g^2 N}{16\pi^2},$$

obtained in ref. [7]. Furthermore, not only the divergent part of the effective action picks up unphysical contributions. Five and higher point effective vertices, that are already finite at one loop in the original theory, get unphysical corrections which remain after the cut-off removal. Then the results differ from those obtained in the original theory. The difference between both results does coincide with the corresponding term of $\log\det^{1/2}(-D^2)$.

The fact that the specific prescription given by Faddeev-Slavnov (with Pauli-Villars determinants computed in the covariant Landau gauge) is not correct does not mean, however, that any method of regularizing gauge theories by high covariant derivatives with Pauli-Villars is necessarily inconsistent. Once we have learned the origin of the problem, it is possible to implement a correct regularization based on similar ideas.

It is in fact fairly easy to modify the prescription to get a consistent regularization method. It is enough to modify the Pauli-Villars contribution defining the new regularized partition function by

$$\mathcal{Z}_\Lambda^{\mathrm{new}} = \int \prod_x dA(x) \exp\left\{-S_\Lambda(A) - \frac{1}{2\alpha_0}\int d^4x\, \partial^\mu A_\mu^a (I - \partial^2/\Lambda^2)^n \partial^\nu A_\nu^a\right\}$$

$$\det\,(-\partial^\mu D_\mu) \prod_j \det{}^{s_j/2}(-D^2) \det{}^{-s_j/2} \mathcal{Q}^L_{m_j} \det{}^{s_j/2}(m_j^2\Lambda^2 - D^2). \qquad (2.10)$$

The partition function $\mathcal{Z}^{\rm new}_\Lambda$ gives rise to finite results for all n-point functions at one loop order provided the Pauli-Villars condition (2.7) is satisfied. Once again to properly define the regularized theory it is necessary to introduce an auxiliary regulator. In appendix A we show that within dimensional regularization the condition (2.7) guarantees the finiteness of the theory. On the other hand the regularization does not generate any non physical correction in the ultraviolet limit and maintains gauge invariance. It is a trivial exercise to see that the computation of the radiative corrections for the effective action at one loop, using this regularization, gives rise to the correct values for the β-function and anomalous dimensions of the gauge field (see Section 5).

In this section we have pointed out the origin of the anomalous result for the beta function and a way of overcoming the problem with a minor change of the prescriptions of the regularization. In order to gain a deeper understanding of the regularization procedure we will examine, in next section, the situation when we use covariant α-gauge instead of covariant Landau gauge to compute the Pauli-Villars determinants. We shall find that in such a case, the straightforward implementation of the Faddeev-Slavnov prescription [3] provides a consistent regularization of the theory. In fact, the simple addition of a mass term to all new Pauli-Villars fields is enough to define a regularization which is free of unphysical contributions.

3 Pauli-Villars regularization in α-gauge

Although the method of Pauli-Villars high covariant derivatives regularization was formulated for an arbitrary α–gauge in the seminal Slavnov paper [3], only the Landau gauge ($\alpha = 0$) case has been considered so far in the literature. Let us analyse the general case in some detail.

It is easy to generalize the Slavnov arguments to show that the one loop divergences of the original Yang-Mills theory, obtained from (2.2) or (2.3), are the same that those of

$$\det(-D^2) \det{}^{-1/2} \mathcal{Q}_{0,\alpha} \det{}^{1/2}(I - D^2/\Lambda^2)^n \qquad (3.1)$$

where

$$\det{}^{-1/2}\mathcal{Q}_{0,\alpha} = \int \prod_x dq(x)\ \exp\Big\{ -\frac{1}{2}\int d^4x d^4y\, q^a_\mu(x) \frac{\delta^2 S_\Lambda}{\delta A^a_\mu(x)\delta A^b_\nu(y)} q^b_\nu(y) \\ -\frac{1}{2\alpha}\int dx D^\mu q^a_\mu (I - D^2/\Lambda^2)^n D^\nu q^a_\nu \Big\}.$$

To show this let us perform the change of variables $q \to (q^\perp, \phi)$ defined in (2.9). Taking into account the Jacobian of the transformation $\det^{1/2}(-D^2)$, we obtain

$$
\begin{aligned}
\det{}^{-1/2}\mathcal{Q}_{0,\alpha} = & \det{}^{1/2}(-D^2) \\
& \cdot \int \prod_x dq^\perp(x) d\phi(x) \exp\Big\{ -\frac{1}{2\alpha}\int dx D^2\phi^a(x)(I - D^2/\Lambda^2)^n D^2\phi^a(x) \\
& -\frac{1}{2}\int d^4x d^4y (q_\mu^{\perp a} + [D_\mu\phi]^a)(x) \frac{\delta^2 S_\Lambda}{\delta A_\mu^a(x)\delta A_\nu^b(y)} (q_\nu^{\perp b} + [D_\nu\phi]^b)(y) \Big\}.
\end{aligned} \tag{3.2}
$$

Now, since S_Λ is gauge invariant,

$$
\int dx \frac{\delta S_\Lambda}{\delta A_\mu^a(x)} [D_\mu\phi]^a(x) = 0,
$$

and

$$
\int dx dy \frac{\delta^2 S_\Lambda}{\delta A_\mu^a(x)\delta A_\nu^b(y)} [D_\mu\phi]^a(x) q_\nu^b(y) = -\int dx \frac{\delta S_\Lambda}{\delta A_\mu^a(x)} f^a_{bc}\phi^c(x) q_\nu^b(x). \tag{3.3}
$$

Then, the vertex involving ϕ fields in the second term of the exponent of (3.2) can be replaced by the right hand side of (3.3). This implies that all space-time derivatives contained in such vertices act on the external fields A_μ instead of the quantum fields $q^\perp$ and ϕ. This means that the corresponding diagrams are not divergent if $n \geq 1$. Such a property is based on the same physical reason that implied the finiteness of the one loop corrections to the ghost-gluon interactions.

Therefore, the divergent contribution of $\det^{-1/2}\mathcal{Q}_{0,\alpha}$ is given by

$$
\begin{aligned}
& \det{}^{-1/2}(D^2)\det{}^{-1/2}(I - D^2/\Lambda^2)^n \\
& \cdot \int \prod_x dq^\perp(x) \exp\Big\{ -\frac{1}{2}\int d^4x d^4y \, q_\mu^{\perp a}(x) \frac{\delta^2 S_\Lambda}{\delta A_\mu^a(x)\delta A_\nu^b(y)} q_\nu^{\perp b}(y) \Big\} \\
& = \det{}^{-1/2}\mathcal{Q}_m^L \det{}^{-1/2}(I - D^2/\Lambda^2)^n
\end{aligned}
$$

and inserting the last expression for the divergences of $\det^{-1/2}\mathcal{Q}_{0,\alpha}$ into (3.1) we obtain the expression (2.3).

Once we have shown that divergences of (3.1) are equal to those of the original theory at one loop we can use this last expression for the Pauli-Villars determinants. Let us introduce

$$
\begin{aligned}
\det{}^{-1/2}\mathcal{Q}_{m,\alpha} = & \int \prod_x dq(x) \exp\Big\{ -\frac{1}{2}\int d^4x d^4y \, q_\mu^a(x) \frac{\delta^2 S_\Lambda}{\delta A_\mu^a(x)\delta A_\nu^b(y)} q_\nu^b(y) \\
& -\frac{1}{2\alpha}\int d^4x \, D^\mu q_\mu^a (I - D^2/\Lambda^2)^n D^\nu q_\nu^a - \frac{1}{2}m^2\Lambda^2 \int d^4x \, q^2 \Big\}.
\end{aligned} \tag{3.4}
$$

One loop divergences can then be cancelled by introducing a suitable number of massive Pauli-Villars determinants. In fact, the regularized theory can be defined from the partition

function

$$\mathcal{Z}_\alpha = \int \prod_x dA(x) \exp\Big\{-S_\Lambda(A) - \frac{1}{2\alpha_0}\int d^4x\, \partial^\mu A^a_\mu (I-\partial^2/\Lambda^2)^n \partial^\mu A^a_\mu\Big\} \tag{3.4}$$

$$\cdot \det\,(-\partial^\mu D_\mu) \prod_j \det{}^{-s_j/2} \mathcal{Q}_{m_j,\alpha_j} \det{}^{s_j}(m_j^2\Lambda^2 - D^2) \det{}^{s_j/2}(I - D^2/\Lambda^2)^n.$$

From the previous analysis of divergences we conclude that $\mathcal{Z}_\alpha$ has, at one-loop, finite effective action provided the s_j are chosen to satisfy the Pauli-Villars condition

$$\sum_j s_j + 1 = 0.$$

In appendix A we verify that this condition it is enough to cancel the $1/\epsilon$ divergences which appear in the different one-loop diagrams by using dimensional regularization as auxiliary cut-off. Notice, again, that due to the special choice of the Pauli-Villars regulators there are not constraints on α_0, α_j and m_j. On the other hand BRST invariance at one-loop is preserved because the new Pauli-Villars regulators are explicitly gauge invariant.

Now it remains to check that the determinants we added to the theory give a contribution independent of the gauge field when the cutoff is removed. If $\alpha \neq 0$ we have

$$\begin{aligned}\lim_{\Lambda\to\infty} \det{}^{-1/2}\mathcal{Q}_{m,\alpha} &= \lim_{\Lambda\to\infty}\int \prod_x dq(x) \exp\Big\{ - \frac{1}{2\Lambda^2}\int d^4x d^4y\, q^a_\mu(x) \frac{\delta^2 S_\Lambda}{\delta A^a_\mu(x)\delta A^b_\nu(y)} q^b_\nu(y) \\ &\qquad - \frac{1}{2\alpha\Lambda^2}\int d^4x\, D^\mu q^a_\mu (I - D^2/\Lambda^2)^n D^\nu q^a_\nu - \frac{1}{2}m^2 \int d^4x\, q^2\Big\} \\ &= \int \prod_x dq(x) \exp\Big\{ -\frac{1}{2}m^2\int d^4x\, q^2\Big\},\end{aligned} \tag{3.5}$$

where the previous equalities are to be understood up to normalization constants. We obtain, then, the desired result since the last expression is a constant independent of A.

We remark that in the limit in which the gauge fixing parameters α_j vanish we get a different theory. Indeed, if we absorb α into the Pauli-Villars fields q, the determinant $\mathcal{Q}_{m,\alpha}$ becomes

$$\begin{aligned}\det{}^{-1/2}\mathcal{Q}_{m,\alpha} = \int \prod_x dq(x)\ \exp\Big\{ &-\frac{\alpha}{2}\int d^4x d^4y\, q^a_\mu(x)\frac{\delta^2 S_\Lambda}{\delta A^a_\mu(x)\delta A^b_\nu(y)} q^b_\nu(y) \\ &- \frac{1}{2}\int d^4x\, D^\mu q^a_\mu (I - D^2/\Lambda^2)^n D^\nu q^a_\nu - \frac{\alpha m^2\Lambda^2}{2}\int d^4x\ q^2\Big\}.\end{aligned}$$

Therefore, if we take the limit $\alpha \to 0$ before removing the regulator Λ we get

$$\det{}^{-1/2}\mathcal{Q}_{m,0} = \int \prod_x dq(x)\ \exp\Big\{ -\frac{1}{2}\int d^4x\, D^\mu q_\mu (I - D^2/\Lambda^2)^n D^\nu q_\nu\Big\}$$

$$= \det{}^{-1/2}(-D^2)\det{}^{-1/2}(I - D^2/\Lambda^2)^n. \tag{3.6}$$

Now, although the second factor in the last line of (3.6) converges to the unity in the limit $\Lambda \to \infty$, the first one gives an additional contribution that indeed coincides with that we already found in the previous section when we analyzed the regularization in covariant Landau gauge. Therefore, in this case we do no get a regularization of pure Yang-Mills theory but a new theory which includes a couple of scalar ghosts fields (one real bosonic, the other complex grassmannian) interacting with the gauge field in the adjoint representation.

The same fact can be seen from a pure perturbative point of view. Indeed, let us consider the propagator of the Pauli-Villars fields,

$$\delta^{ab}\left[\frac{1}{p^2(1+p^2/\Lambda^2)^n + m^2\Lambda^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \frac{\alpha}{p^2(1+p^2/\Lambda^2)^n + \alpha m^2\Lambda^2}\frac{p_\mu p_\nu}{p^2}\right].$$

If $\alpha \neq 0$ one loop diagrams with a Pauli-Villars field running around the loop and negative degree of divergence, like five or higher point functions, vanish in the limit $\Lambda \to \infty$. Then, the new determinants do not give any contribution to the finite parts of the theory. However when we take the limit $\alpha \to 0$ before removing the cutoff Λ we see that although the longitudinal part of the propagator, that behaves like

$$\frac{\alpha}{p^2(1+p^2/\Lambda^2)^n}\cdot\frac{p_\mu p_\nu}{p^2},$$

seems to give a null contribution in this limit, this is not true since there are also vertices with a factor α^{-1}, coming from the gauge fixing term. It is easy to show that divergences of these vertices in $\alpha = 0$ cancel out with the zeros of the propagators and they contribute to a finite result (independent of m) that does not vanish when we remove the cutoff. It can be shown, again, that the new contributions are exactly those of $\det^{1/2}(-D^2)$.

This singular character of Landau gauge was already noticed in another context by Kennedy and King [14]. In our case it means that standard Slavnov regularization prescription is correct in any covariant α-gauge for the Pauli-Villars fields except for Landau gauge $\alpha = 0$. In the latter case, the prescription has to be modified along the lines described in previous section.

4 Overlapping Divergences

So far we have considered the regularization of the theory at one loop. One could argue that, after adding high derivatives, this is all we must consider because diagrams of more than one loop are, then, finite by power counting and only one loop subdiagrams diverge. This is true, but among those subdiagrams there are the mixed loops of A lines and q lines that diverge and their divergences are not cancelled by any other contribution. This problem, known as

the overlapping divergence problem, has remained for a long time as the main obstruction to the implementation of the high covariant derivatives method for non-abelian gauge theories as proposed in Ref. [4].

There have been several proposals to overcome this problem but not all of them have been complete. In Ref. [15] it was suggested to use different actions $S_{\Lambda,j}$ with less covariant derivatives than the original S_Λ in Pauli-Villars determinants $\mathcal{Q}^L_{m_j}$. Explicitly, if we take

$$S_{\Lambda,j}(A) = \frac{1}{4g^2}\int d^4x F^a_{\mu\nu}[(I+\Delta_{\lambda_j}/\Lambda^2)^{n_j}]^{a'\mu}_{a\ \mu'} F^{\mu'\nu}_{a'} \qquad \text{with} \qquad 2 \leq n_j < n-1, \tag{4.1}$$

and use $S_{\Lambda,j}$ instead of S_Λ in (2.5) and (3.4) we see by power counting that the overlapping divergences are absent. If in addition we modify Faddeev-Slavnov prescription for covariant Landau gauge as in Sec. 2, or use α-gauge for the Pauli-Villars determinants as in Sec. 3, we can, at least in principle, define a theory regularized at any order and free of unphysical contributions. The problem, however, is that in order to ensure finiteness of the theory at one loop one has to change the Pauli-Villars condition (2.7), and even the way to determine the new conditions requires a more subtle analysis.

As mentioned before, the complete analysis of the mechanism of cancellation of one-loop divergences requires the introduction of an auxiliary regulator to handle the different divergences which appear by power counting [13][5][7]. The divergences of the different diagrams are computed in the appendix by means of a dimensional regularization as auxiliary regulator. The results show that the one loop effective action is finite if the condition

$$\sum_j s_j[\lambda_j^2(5n_j^2 - 18n_j + 16) - \lambda_j(4n_j^2 + 10n_j + 4) + 4n_j^2 + 5n_j - 7/3] = 0 \tag{4.2}$$

is satisfied. Here and below $n_0 = n$ and $s_0 = 1$ and we assume $n_j \geq 2$.

Another possible choice of the auxiliary regularization is to introduce a momentum cut-off on one internal momentum in every loop [13] or still better, in order to have an unambiguous definition, a momentum cut-off in *all* internal lines [5]. In general, this kind of auxiliary regularization does not preserve gauge invariance, but has the advantage of having a non perturbative meaning [5]. On the other hand as we shall discuss below the breaking of gauge invariance can be controlled and it is possible to tune the parameters of the regularization in order to actually have a fully gauge invariant regularized theory.

If we use momentum cut-off as auxiliary regulator the cancellation conditions become more stringent. In Landau gauge all one-loop divergences cancel out if the following conditions are satisfied

$$\sum_j s_j(n_j+1)^2 + \frac{1}{6}s_0 = 0, \tag{4.3a}$$

$$\sum_{j\neq 0} s_j = 0, \tag{4.3b}$$

$$\sum_j s_j[n_j + \frac{2}{3}] = 0, \tag{4.3c}$$

$$\sum_j s_j[\lambda_j^2(5n_j^2 - 18n_j + 16) - \lambda_j\,(4n_j^2 + 10n_j + 4) + 4n_j^2 + 5n_j - 7/3] = 0. \tag{4.3d}$$

Conditions (4.3a) and (4.3b) are necessary in order to cancel linear divergences, (4.3c) and (4.3d) stand for the cancelation of quadratic and logarithmic divergences, respectively. The existence of linear divergences is a consequence of our way of taking the momentum cut-off in *all* internal lines and they may not appear for other choices of auxiliary regulator. Quadratic and linear divergences are absent for gauge invariant pre-regularizations, while the logarithmic ones are universal. Note that if all n_j's and λ_j's were taken equal the last two conditions would reduce to (2.7).

Although the sharp momentum cut-off regularizations break gauge invariance, one could think that when the pre-regulator is removed gauge invariance will be restored, because the theory is finite. Unfortunately this is wrong in general, and Slavnov-Taylor identities are violated by finite contributions generated by the momentum cutoff. The anomalous contribution also appears as extra terms in one-loop diagram identities of the diagrammatic approach [16]. Therefore one needs to impose some additional constraints on the regulating parameters in order to cancel such anomalous contributions to Slavnov-Taylor identities [17]. Indeed, although in principle gauge invariance is lost due to the auxiliary momentum cut-off it is possible to adjust the free parameters of the regularization in order to recover it. Only a finite number of diagrams are responsible for the breaking of the symmetry i.e one loop diagrams with two, three or four external legs. Thus, we only need to compute the finite parts of these diagrams and impose more constraints on the parameters of the regularization in order to ensure that the terms that do not fulfil Ward identities cancell out.

The explicit calculation of these additional conditions, though finite, is lengthy and tedious and has not been carried out yet in four-dimensional gauge theories. In three-dimensional space-times, an explicit choice of parameters that preserve gauge invariance of the regulated theory has been found in [17]. Once all these requirements are fulfilled we obtain a consistent gauge invariant regularization which can be used for cases where more conventional regularizations fail to give an appropriate description of the physical effects.

5 Renormalization Group Coefficients.

In order to confirm the consistency of the regularization method introduced in previous section, we calculate here the one loop renormalization group coefficients and we shall verify that the results do coincide with the universal values for the β-function and the anomalous

dimension of the gauge field. These coefficients can be extracted from the divergent terms of the effective action in the limit $\Lambda \to \infty$.

We shall use the regularization scheme introduced in section 4 with both gluons and Pauli-Villars fields in α gauge, i.e. we shall take expression (3.4) with S_Λ replaced by $S_{\Lambda,j}$ in the computation of $\det^{1/2}\mathcal{Q}_{m_j,\alpha_j}$ and n replaced by n_j in $\det^{s_j/2}(I - D^2/\Lambda^2)^n$.

Taking dimensional regularization as auxiliary regularization, we obtain finite results for $D = 4$ and $\Lambda < \infty$, provided (4.2) is fulfilled.

Of course when the ultraviolet regulator Λ is removed divergences appear. In this case we shall have only logarithmic divergences as these are the only ones allowed by gauge invariance. The computation of the $\log\Lambda$ terms, which are relevant for γ and β coefficients of the renormalization group, can be considerably simplified by the following observation. Let us consider the corrections to the vacuum polarization tensor at one loop. The only terms that generate a $\log\Lambda$ contribution are those that were primitively ultraviolet divergent together with those with a degree of divergence -2 in the infrared (when the external momentum goes to zero). The former were already computed to ensure finiteness of the theory and the second ones get contributions from massless fields and only from the part of their vertices with the least number of derivatives. The same is true for three points functions changing the degree of divergence in the infrared to -1. Below we present the different contributions at one loop to gluon and Faddeev-Popov ghost two point functions as well as to the ghost-gluon-ghost effective vertex.

The $\log\Lambda$ contribution to the vacuum polarization tensor coming from loops of gluons in α_0 gauge (diagrams (1) and (2) of Figure 1) is given by:

$$\begin{aligned} {}^{g}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) \;=\; & \frac{g^2N}{16\pi^2}\delta^{a_1a_2}\Big[4n^2+5n+2-\alpha_0-\lambda(4n^2+10n+4) \\ & \qquad +\lambda^2(5n^2-18n+16)\Big]\,(q^2\delta_{\mu_1\mu_2}-q_{\mu_1}q_{\mu_2})\log\Lambda+\mathcal{O}(1) \end{aligned}$$

for $n \geq 2$.

Loops of Faddeev-Popov ghosts do not generate any Λ dependence. But there are two different $\log\Lambda$ contributions from loops of Pauli-Villars fields. That coming from $\det^{-1/2}\mathcal{Q}_{m_j,\alpha_j}$ yields

$$\begin{aligned} {}^{PV}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) \;=\; & \frac{g^2N}{16\pi^2}\delta^{a_1a_2}\Big[4n_j^2+16n_j/3-5/3-\lambda(4n_j^2+10n_j+4) \\ & \qquad +\lambda^2(5n_j^2-18n_j+16)\Big]\,(q^2\delta_{\mu_1\mu_2}-q_{\mu_1}q_{\mu_2})\log\Lambda+\mathcal{O}(1), \end{aligned}$$

where $n_j \geq 2$ is assumed. The corresponding contribution from $\det^{1/2}(m^2\Lambda^2 - D^2)^{n_j}$ reads

$${}^{PV_s}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) = -\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\frac{n_j}{3}\left(q^2\delta_{\mu_1\mu_2}-q_{\mu_1}q_{\mu_2}\right)\log\Lambda+\mathcal{O}(1).$$

Taking into account the finiteness condition (4.2) the total divergent logarithmic contribution of the vacuum polarization tensor becomes

$$\left(\frac{13}{3} - \alpha_0\right) \frac{g^2 N}{16\pi^2} \delta^{a_1 a_2} \left(\delta_{\mu_1 \mu_2} q^2 - q_{\mu_1} q_{\mu_2}\right) \log \Lambda + \mathcal{O}(1), \tag{5.1}$$

and the renormalization group parameter associated to the anomalous dimension of the gauge field,

$$\gamma_A(g) = \left(\frac{13}{6} - \frac{\alpha_0}{2}\right) \frac{g^2 N}{16\pi^2}. \tag{5.2}$$

The ghost self-energy has analogous logarithmic divergent terms

$$-\left(\frac{3}{2} - \frac{\alpha_0}{2}\right) \frac{g^2 N}{16\pi^2} \delta^{a_1 a_2} q^2 \log \Lambda + \mathcal{O}(1) \tag{5.3}$$

which contribute to the renormalization of the anomalous dimension parameter of the ghost field

$$\gamma_c(g) = \left(\frac{3}{4} - \frac{\alpha_0}{4}\right) \frac{g^2 N}{16\pi^2}.$$

Finally, the relevant one-loop contribution to the effective ghost-gluon-ghost vertex obtained from the diagrams (1) and (2) of figure 2 is given by

$$-\, i g \alpha_0 \frac{g^2 N}{16\pi^2} f^{abc} p_\mu \log \Lambda + \mathcal{O}(1). \tag{5.4}$$

The comparison of (5.1), (5.3) and (5.4) gives the beta function of the coupling constant g,

$$\beta(g) = -\frac{11}{3} \frac{g^3 N}{16\pi^2} \tag{5.5}$$

which agrees with the standard (universal) value obtained by other methods [8]. Therefore, one loop results for the β-function and the anomalous dimensions parameters γ_A and γ_c confirm the validity of the high covariant derivatives regularization method.

6 Geometric Regularization.

In this section we shall analyse, for the sake of completeness, the results obtained by geometric regularization method. The method was introduced in Refs. [11], [5] and [6], in order to solve the problems of the conventional high covariant derivative methods and to incorporate to the theory geometric elements of the space of gauge orbits which have a non-perturbative meaning. One of the advantages of the method is that it overcomes the Gribov problem giving a global interpretation to the functional integral measure on the gauge orbit space.

The results listed below were first obtained in Ref. [6] and the reason to include them here is to show the perturbative agreement of all consistent regularizations based on the method of high covariant derivatives.

The calculation is also considerably simplified by the same observation made in the previous section about the connection of logarithmic divergences on Λ with the infrared divergences on the one-loop corrections to the effective action. The relevant contribution of gluon loops to the vacuum polarization tensor $\Pi^{ab}_{\mu\nu}$ is the same as that obtained by the method analysed in the previous section (5.1). However, the calculation of the ghost loops contribution is rather different.

The central idea of geometric regularization is to introduce two types of ghost fields: vector ghosts associated to the metric of the orbit space and scalar ghosts associated to the volume of the fibres of gauge equivalent gauge fields. Scalar ghosts have gauge invariant interactions whereas vector ghosts do not. The sum of their contributions to the gluon effective action does coincide with that of the Faddeev-Popov determinant. The splitting of the Faddeev-Popov determinant into the product of two determinants

$$\det(-\partial^\mu D_\mu) = \det\nolimits_L^{\frac{1}{2}} \left(\delta^\nu_\mu - D_\mu (D^2)^{-1} D^\nu\right) \ \det\nolimits^{\frac{1}{2}}(-D^2) \tag{6.1}$$

is based in the observation made by Babelon and Viallet in Ref. [18]. Here $\det_L$ stands for the determinant restricted to fields in the Landau gauge $\partial_\mu q^\mu = 0$. The geometric interpretation of such a splitting allows to identify the contribution of vector ghosts with the functional riemannian volume of the gauge orbit space, and that of the scalar ghosts with the functional volume of the different gauge orbits. The non gauge invariant character of the former, because of the restriction of $\det_L$ to a non invariant subspace, can be understood as a consequence of the necessary choice of coordinates to parametrize the space of gauge orbits.

The global character of this geometric formulation of the Faddeev-Popov gauge fixing method is preserved by geometric regularization which replaces such a structure by a stronger riemannian structure and two regularizing nuclear structures. Because of the vectorial nature of those ghost fields it is natural to impose on them the same cut-off that to the gauge fields and to interpret it as the restriction to a submanifold of the orbit space. This feature provides a non-perturbative meaning to the regularization method.

Now, since the relevant terms of the vertices are those which are Λ independent, the sum of the contributions generated by nuclear, metric and scalar ghosts reduces to that of the Faddeev-Popov ghosts. This follows from the Babelon and Viallet splitting of Faddeev-Popov determinant and from the theorem that establishes that logarithmic divergences are always preserved by determinant factorizations (see [5]). Therefore, the global one loop corrections to the anomalous dimension of the gauge field γ_A are also given by (5.2) with $\alpha_0 = 0$.

The structure of geometric regularization implies that there are two different types of ghost self-energies. For vector (metric) ghosts the logarithmic divergent contribution is

given by

$$\frac{3}{4}\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\left(\delta_{\mu_1\mu_2}-\frac{q_{\mu_1}q_{\mu_2}}{q^2}\right)\log\frac{\Lambda^2}{q^2}$$

which generates a renormalization of the anomalous dimension parameter of the metric ghost field

$$\gamma_\psi=\frac{3}{4}\frac{g^2N}{16\pi^2}.$$

The remaining scalar ghost fields pick up a logarithmic divergent contribution to its self-energy of the form

$$3\frac{g^2N}{16\pi^2}\delta^{a_1a_2}q^2\log\frac{\Lambda^2}{q^2} \tag{6.2}$$

which yields a non-trivial anomalous dimension parameter

$$\gamma_\varphi=3\frac{g^2N}{16\pi^2}.$$

The renormalization of the coupling constant can be obtained from the computation of the effective coupling of vector or scalar ghosts to the gauge field. The one loop correction to the effective vertex of one gluon and two vector ghosts, coming from diagrams (1)-(5) of figure 3, vanishes, while the first perturbative correction to the effective vertex with two scalar ghosts is given by

$$\frac{9}{4}\frac{g^2N}{16\pi^2}igf^{abc}(2p+q)_\mu\log\frac{\Lambda^2}{q^2}. \tag{6.3}$$

The value of the β-function obtained in both cases

$$\beta=-\frac{11}{3}\frac{g^3N}{16\pi^2}, \tag{6.4}$$

and agrees with the standard value obtained in previous sections.

7 Conclusions.

The previous results show that the high covariant regularization method is a consistent regularization of gauge theories when one loop divergences are properly regularized by means of gauge invariant Pauli-Villars methods.

Because of the vectorial character of the gauge fields the method requires the introduction of Pauli-Villars vector fields. If those vector fields are considered in a generic α-gauge the straightforward generalization of Slavnov method provides a consistent BRST invariant regularization of the theory. However, if those fields are considered in a covariant Landau

gauge ($\alpha = 0$), the Slavnov method is not a proper regularization of the theory. In such a case the method introduces massless gauge fields for each vector Pauli-Villars regulator and those fields do not decouple from gauge particles in the limit when the ultraviolet cut-off is removed. The existence of such a term is due to the singular character of the limit $\alpha \to 0$ which does not commute with the removal of the Pauli-Villars regularization mass parameter. This fact explains the discrepancy found in [7] in the calculation of β and γ-functions using the Slavnov regularization method.

The Pauli-Villars regularization introduced in this paper does not suffer from any of those problems. It is a slight modification of the original Slavnov proposal which solves de problem of Landau gauge so that the limit $\alpha \to 0$ is smooth. The calculation of the β and all γ-functions at one loop order yields the standard universal values and confirms the validity of the regularization method.

On the other hand we have shown how it is fairly easy to overcome the problem of overlapping divergences which is present in the Slavnov proposal [4]. The formal manipulations of the functional integral which show the consistency of the regularization can be substantiated by introducing an auxiliary regulator. Dimensional regularization provides a gauge invariant pre-regulator which can be used to guarantee the consistency of the theory. A momentum cut-off provides also a natural approximation which has a meaning beyond perturbation theory, but it breaks the gauge symmetry. Gauge invariance can be recovered once the spurious contributions generated by the sharp momentum cut-off are cancelled by tuning the parameters of the regulators. From a non-perturbative viewpoint this regularization method is useful in cases where the lattice regularization is not suitable.

Indeed, the simple fact that the problems discussed in this paper have not been addressed earlier shows that the high covariant method has been mainly used so far for formal arguments but not explicit calculations. The only problem where the method proved to be useful was in the the regularization of 2+1 dimensional gauge theories with topological pseudoscalar Chern-Simons term. However, in such a case the extra unphysical contributions did not appear because the main goal was to calculate the radiative corrections to the Chern-Simons coupling constant, and it is obvious that there are not anomalous unphysical corrections at one loop to such an interaction, because of the scalar character of the anomalous factor $\det^{1/2}(-D^2)$. However, pathological contributions will appear at two loops and will provide a non-trivial renormalization of the Chern-Simons coupling constant.

In summary, the high covariant derivative regularization method in the formulation presented here is fully consistent and can be used as an alternative regularization in those cases where other methods fail either in perturbation theory or in the analysis of non-perturbative effects.

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Appendix : Cancellation of one-loop divergences

The special conditions which satisfy the exponents and parameters of the different regulators where chosen to ensure that all one-loop divergent contributions which appear in the different regularized partition function introduced in the paper, cancel out to produce finite effective actions.

In order to clarify the mechanism of cancellation we include in this appendix the explicit calculations, using the dimensional regularization as auxiliary regulator.

1. One loop divergent contributions to the vacuum polarization tensor

1.1 Contribution of gluon loops.

In Landau gauge the divergent contribution of gluon loops with regularized action $S_\Lambda(A)$ is given by (diagrams (1) and (2) of Figure 1)

$$
\begin{aligned}
{}^{g}\Pi^{a_1 a_2}_{\mu_1\mu_2}(q) \;=\; -\frac{1}{\epsilon}\frac{g^2 N}{16\pi^2}\delta^{a_1 a_2}\Big[\Big(&4n^2+5n-7/3-\lambda(4n^2+10n+4)\\
&+\lambda^2(5n^2-18n+16)\Big)(q^2\delta_{\mu_1\mu_2}-q_{\mu_1}q_{\mu_2})\\
&+\frac{1}{6}q^2\delta_{\mu_1\mu_2}+\frac{1}{3}q_{\mu_1}q_{\mu_2}\Big]+\mathcal{O}(1)
\end{aligned}
$$

for $n\geq 2$. In the case $n<2$ such a contribution cannot be obtained by taking $n=1$ or 0 in the previous expression. This fact can be easily understood if we look closely at the vertices of diagram (1) in Figure 1. For large n there is a particular term in these vertices that generates divergences in the vacuum polarization tensor and comes from taking as external line a gluon field from $F_{\mu\nu}$, say the one on the right of (2.1), and as the two internal lines one from the $F_{\mu\nu}$ on the left and the other from the most right Laplace-Beltrami operator. Note that if we take the third A-field from any other Laplace-Beltrami operator we do not have a divergence in the two point function, therefore the contribution of this part of the vertex to the ϵ-divergent piece of ${}^{g}\Pi^{a_1 a_2}_{\mu_1\mu_2}(q)$ does not depend on n provided $n\geq 1$ but it is absent if $n=0$. For $n=1$ there is a similar phenomenon in the four point vertex.

The same results hold for the theory in α_0-gauge (2.6). In this case the extra α_0 divergent contributions arising in each diagram of Figure 1 cancel out in the sum of both diagrams provided $n\geq 2$. Such a cancellation does not occur for $n=0$.

1.2 Contribution of Faddeev-Popov ghost loops.

It is given by

$$ {}^{\phi\pi}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) = \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\left(\frac{1}{6}q^2\delta_{\mu_1\mu_2} + \frac{1}{3}q_{\mu_1}q_{\mu_2}\right) + \mathcal{O}(1) $$

1.3 Contribution of Pauli-Villars regulators.

There are two different contributions. That generated by $\det^{-1/2}\mathcal{Q}^L_m$, which is given by

$$ \begin{aligned} {}^{PV}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) \;=\; & -\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\Big[4n^2 + 5n - 5/3 + \lambda(4n^2+10n+4) \\ & - \lambda^2(5n^2-18n+16)\Big]\left(q^2\delta_{\mu_1\mu_2} - q_{\mu_1}q_{\mu_2}\right) + \mathcal{O}(1), \end{aligned} $$

for $n \geq 2$. And that induced by $\det^{1/2}(m^2\Lambda^2 - D^2)\det^{1/2}(-D^2)$, which reads

$$ {}^{PV_s}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) = \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\frac{2}{3}\left(q^2\delta_{\mu_1\mu_2} - q_{\mu_1}q_{\mu_2}\right) + \mathcal{O}(1) $$

and is the same that that of $\det(m^2\Lambda^2 - D^2)$.

If we consider the Pauli-Villars fields associated to $\det^{-1/2}\mathcal{Q}_{m,\alpha}$ we get the following divergent contribution

$$ \begin{aligned} {}^{PV}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) \;=\; & -\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\Big[4n^2 + 16n/3 - 5/3 - \lambda(4n^2+10n+4) \\ & + \lambda^2(5n^2-18n+16)\Big]\left(q^2\delta_{\mu_1\mu_2} - q_{\mu_1}q_{\mu_2}\right) + \mathcal{O}(1), \end{aligned} $$

for $n \geq 2$. Note that, as it was expected from the considerations of section 3, the α-dependent divergences generated by diagrams (1) and (2) cancel out.

Finally, the contribution from $\det^{1/2}(m^2\Lambda^2 - D^2)^n$ is

$$ {}^{PV_s}\Pi^{a_1a_2}_{\mu_1\mu_2}(q) = \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\delta^{a_1a_2}\frac{n}{3}\left(q^2\delta_{\mu_1\mu_2} - q_{\mu_1}q_{\mu_2}\right) + \mathcal{O}(1). $$

2. One loop divergent contributions to the three-point function

2.1 Contribution of gluon loops

In Landau gauge the divergent contribution of gluon loops with regularized action $S_\Lambda(A)$ is given for $n \geq 2$ by

$$ \begin{aligned} {}^{g}\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) \;=\; & \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\Big[4n^2 + 5n - 9/4 + \lambda^2(5n^2-18n+16) \\ & -\lambda(4n^2+10n+4)\;\Big]\,\Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} + \mathcal{O}(1), \end{aligned} $$

where

$$ \Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} = igf^{a_1a_2a_3}\left[(q_1-q_2)_{\mu_3}\delta_{\mu_1\mu_2} + (q_2-q_3)_{\mu_1}\delta_{\mu_2\mu_3} + (q_3-q_1)_{\mu_2}\delta_{\mu_1\mu_3}\right]. $$

for $n \geq 2$. As before, the same results hold for the theory in α_0 gauge defined in (2.6).

2.2 Contribution of Faddeev-Popov ghost loops

It is given by

$$ {}^{\phi\pi}\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) = -\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\frac{1}{12}\Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} + \mathcal{O}(1). $$

2.3 Contribution of Pauli-Villars regulators

There are two different contributions. That induced by to $\det^{-1/2}\mathcal{Q}^L_m$ which is given by

$$ \begin{aligned} {}^{PV}\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) \;=\; & \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\Big[4n^2+5n-5/3-\lambda(4n^2+10n+4) \\ & + \lambda^2(5n^2-18n+16)\Big]\,\Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} + \mathcal{O}(1), \end{aligned} $$

with $n\geq 2$, and that generated by $\det^{1/2}(m^2\Lambda^2-D^2)\det^{1/2}(-D^2)$ which reads,

$$ {}^{PV_s}\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) = -\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\frac{2}{3}\Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} + \mathcal{O}(1), $$

and is the same that that of $\det(m^2\Lambda^2-D^2)$.

If we consider the Pauli-Villars fields associated to $\det^{-1/2}\mathcal{Q}_{m,\alpha}$ we get the following divergent contribution

$$ \begin{aligned} {}^{PV}\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) \;=\; & \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\Big[4n^2+16n/3-5/3-\lambda(4n^2+10n+4) \\ & +\lambda^2(5n^2-18n+16)\Big]\,\Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} + \mathcal{O}(1), \end{aligned} $$

for $n\geq 2$ and once again the α-dependent divergences generated by the different diagrams cancel each other.

Finally, the contribution from $\det^{1/2}(m^2\Lambda^2-D^2)^n$ is

$$ {}^{PV_s}\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) = -\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\frac{n}{3}\Psi^{a_1a_2a_3}_{\mu_1\mu_2\mu_3} + \mathcal{O}(1). $$

3. One loop divergent contributions to the four-point function

3.1 Contribution of gluon loops

In Landau gauge the divergent contribution of gluon loops with regularized action $S_\Lambda(A)$ is given by

$$ \begin{aligned} {}^{g}\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3,q_4) \;=\; & \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\Big[\,\Big(4n^2+5n-7/3-\lambda(4n^2+10n+4) \\ & +\lambda^2(5n^2-18n+16)\,\Big)\,\Theta^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4} + \frac{1}{6}\Sigma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}\Big] + \mathcal{O}(1), \end{aligned} $$

for $n\geq 2$, where

$$ \begin{aligned} \Theta^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4} \;=\; -\big[\; & f^{a_1a_2c}f^{a_3a_4c}(\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}-\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3}) \\ & +f^{a_1a_3c}f^{a_2a_4c}(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}-\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3}) \end{aligned} $$

$$+f^{a_1a_4c}f^{a_3a_2c}(\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}-\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4})]$$

and

$$\begin{aligned}\Sigma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4} &= \frac{1}{N}(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}+\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}+\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3})\,(f^{\alpha_1a_1\alpha_2}f^{\alpha_2a_2\alpha_3}f^{\alpha_3a_3\alpha_4}f^{\alpha_4a_4\alpha_1}\\ &\quad +f^{\alpha_1a_1\alpha_2}f^{\alpha_2a_2\alpha_3}f^{\alpha_3a_4\alpha_4}f^{\alpha_4a_3\alpha_1}+f^{\alpha_1a_1\alpha_2}f^{\alpha_2a_3\alpha_3}f^{\alpha_3a_2\alpha_4}f^{\alpha_4a_4\alpha_1})\end{aligned}$$

For $n\le 1$ the contribution is different for the reasons mentioned above. As before, the same results hold for gluons in α_0 gauge.

3.2 Contribution of Faddeev-Popov ghost loops

It is given by

$$^{\phi\pi}\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3,q_4)=-\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\frac{1}{6}\Sigma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}+\mathcal{O}(1).$$

3.3 Contribution of Pauli-Villars regulators

There are two different contributions. That generated by $\det^{-1/2}\mathcal{Q}^L_m$ which is given by

$$\begin{aligned}^{PV}\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3,q_4) &= \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\Big[4n^2+5n-5/3-\lambda(4n^2+10n+4)\\ &\quad +\lambda^2(5n^2-18n+16)\Big]\,\Theta^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}+\mathcal{O}(1),\end{aligned}$$

where $n\ge 2$, and the contribution form $\det^{1/2}(m^2\Lambda^2-D^2)\det^{1/2}(-D^2)$ which reads,

$$^{PV_s}\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3,q_4)=-\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\frac{2}{3}\Theta^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}+\mathcal{O}(1),$$

and is the same that that of $\det(m^2\Lambda^2-D^2)$.

If we consider the Pauli-Villars fields given by $\det^{-1/2}\mathcal{Q}_{m,\alpha}$ we get the following divergent contribution

$$\begin{aligned}^{PV}\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3,q_4) &= \frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\Big[4n^2+16n/3-5/3-\lambda(4n^2+10n+4)\\ &\quad +\lambda^2(5n^2-18n+16)\Big]\,\Theta^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}+\mathcal{O}(1),\end{aligned}$$

for $n\ge 2$ and again the α-dependent divergences generated by the different diagrams cancel each other.

Finally, the contribution from $\det^{1/2}(m^2\Lambda^2-D^2)^n$ is

$$^{PV_s}\Gamma^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(q_1,q_2,q_3,q_4)=-\frac{1}{\epsilon}\frac{g^2N}{16\pi^2}\frac{n}{3}\Theta^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}+\mathcal{O}(1).$$

In this paper we have analyzed two regularization schemes. The first one is that of sections 2 and 3 where we do not care about overlapping divergencies and the number of high derivatives in gluons and Pauli-Villars fields is the same. In such a case the cancellation

of all $1/\epsilon$ divergence follows from condition (2.7) which is enough to guarantee the finiteness of the regularized theory in the limit $\epsilon \to 0$.

In the second scenario, introduced in section 4 to cure the overlapping divergences problem, the different Pauli-Villars fields must have different number of derivatives and then the condition for cancellation of $1/\epsilon$ divergences reduces to (4.2).

In conclusion, in both cases the cancellation of divergences is not only a consequence of the formal identities derived from functional integral methods, but it is a fact once we introduce an appropriate auxiliary regulator.

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Figure Captions

Figure 1: Radiative corrections to the vacuum polarization involving gluon loops.

Figure 2: One loop contributions to the 3-vertex interaction of gluons with Faddeev-Popov ghosts.

Figure 3: One loop contributions to the 3-vertex interaction of gluons with scalar ghosts in geometric regularization.

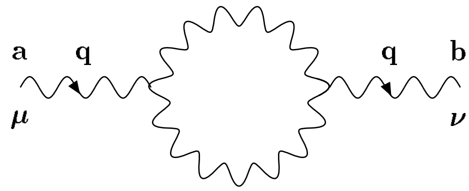


Diagram (1)

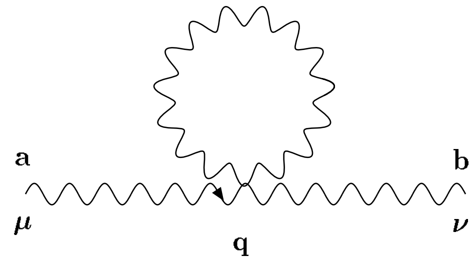
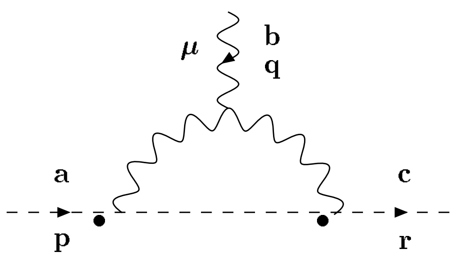


Diagram (2)

FIGURE 1



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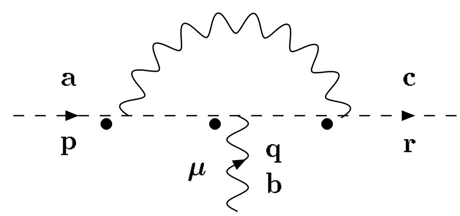
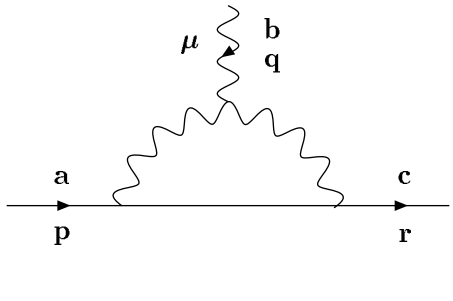


Diagram (2)

FIGURE 2



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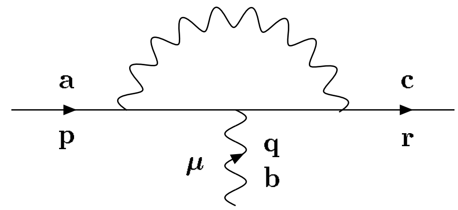

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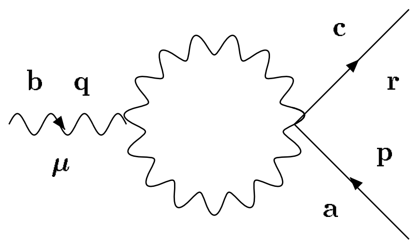

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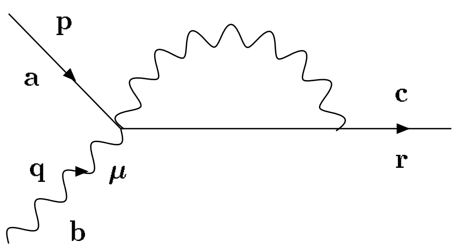

Diagram (4)

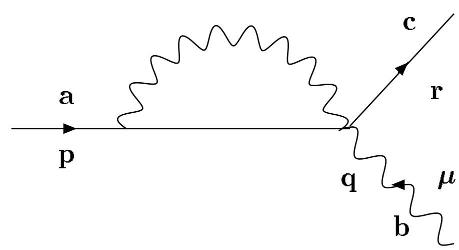

Diagram (5)

FIGURE 3